Hashing
Data Distribution

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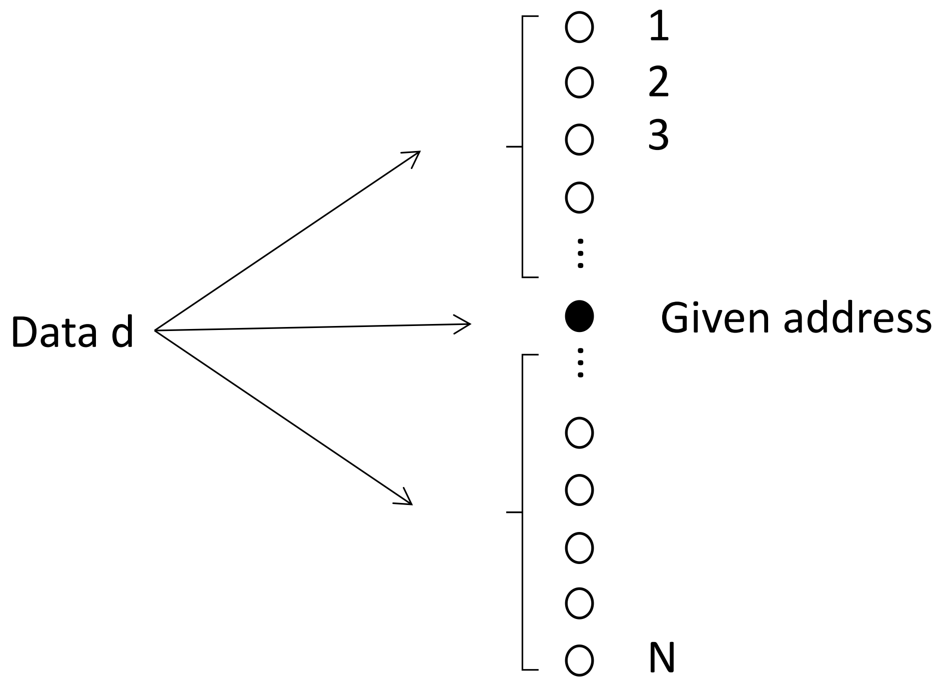
Hashing / Data Distribution

Poisson Distribution

Suppose

- N possible addresses
- A given address
- Data d to hash



Let’s consider the two events:
A : A given address is not selected
B : A given address is selected

When data is hashed, one of the events (A or B) occurs for a given address.

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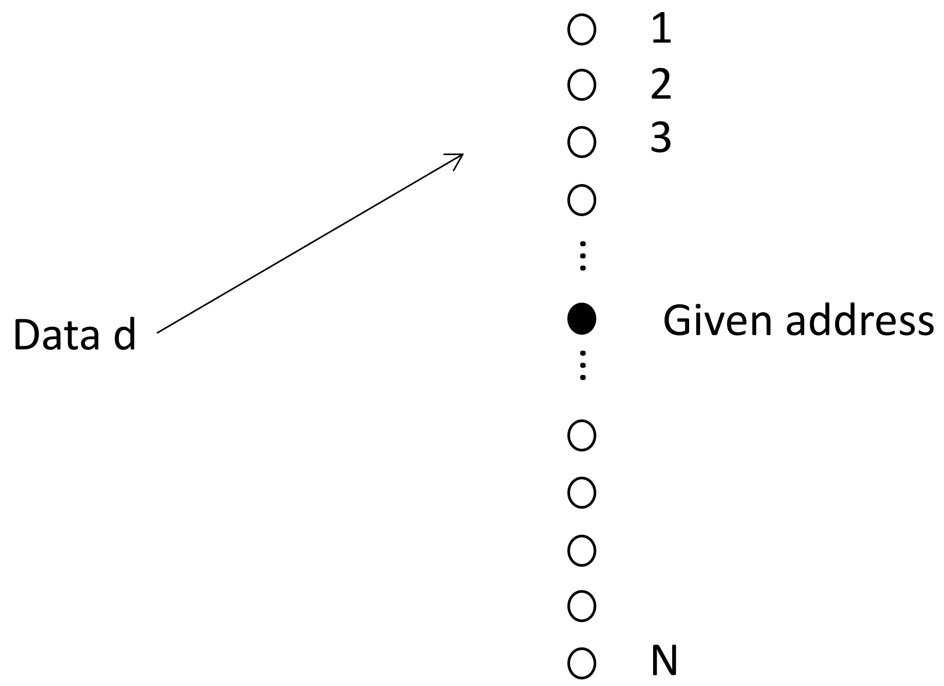

What is the probability that a given address is not chosen?

Event : A

P(A) = a = 1 - 1/N = (N-1)/N= a
If N=10 then a=0.9

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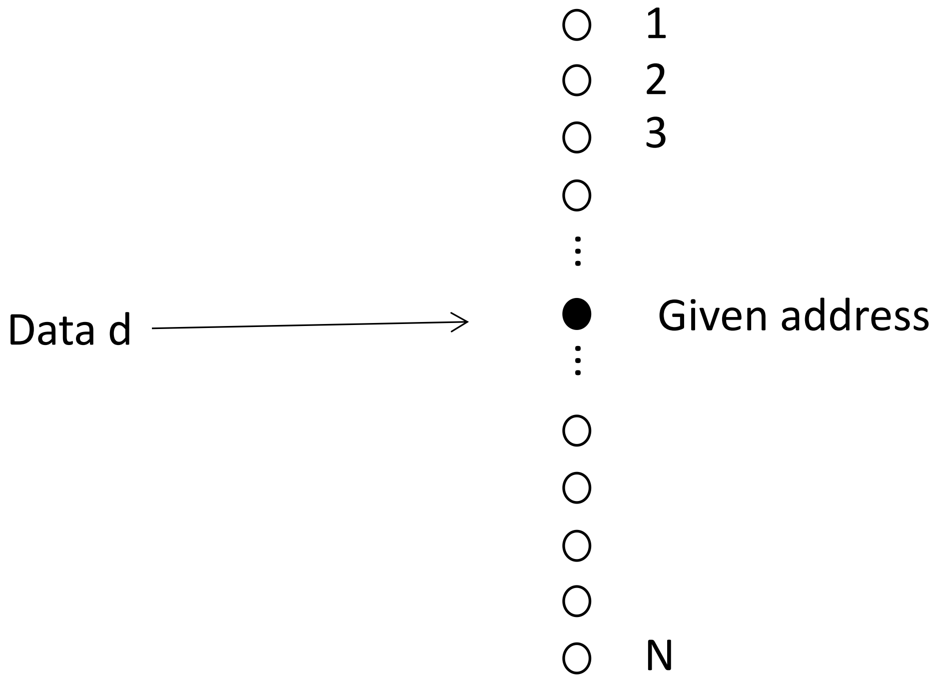

What is the probability that a given address is chosen?

Event : B

P(B) = b = 1/N

If N=10 then a=0.1

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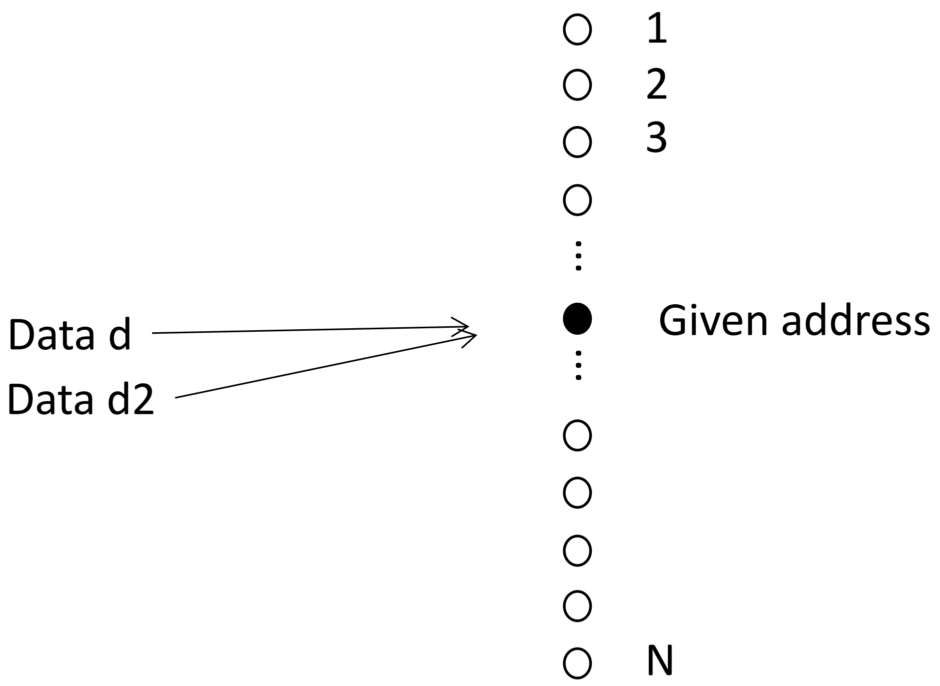

What is the probability that two data hash to the same address?

Event : B B

P(BB) = b.b = 1/N . 1/N (Independent events)

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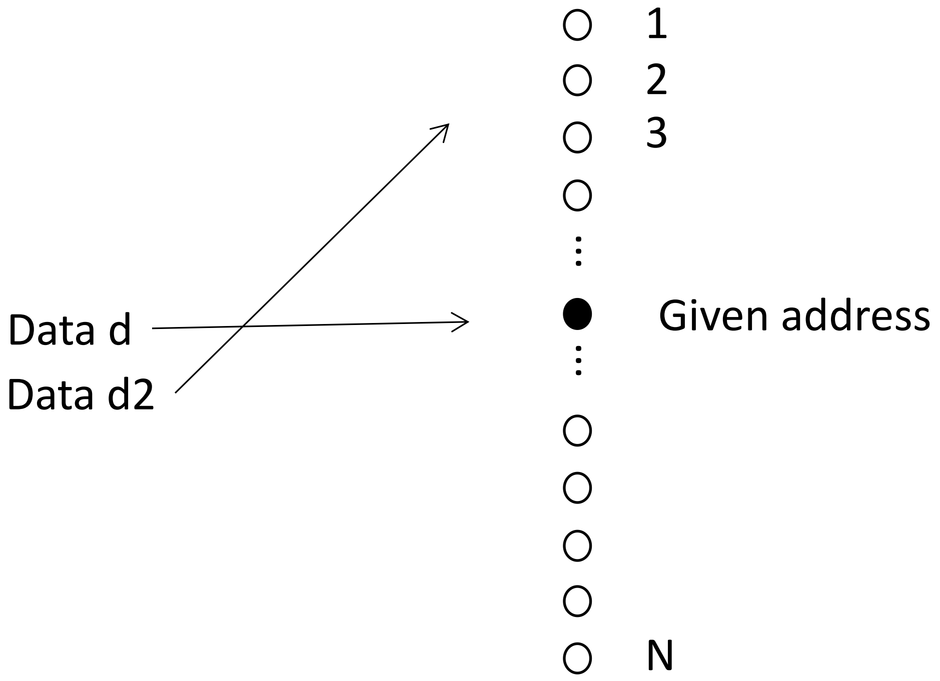

What is the probability that the first data is hashed to the given address and the second data is hashed to an address different from the first one?

Event : B A

P(BA) = b.a = 1/N . (N-1/N)

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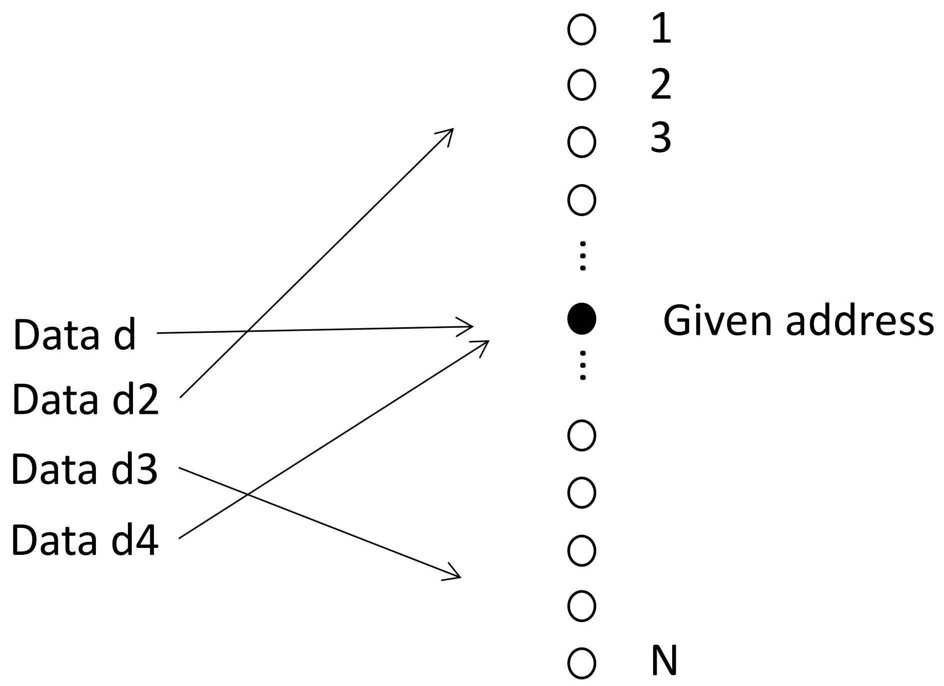

What is the probability associated with the event BAAB?

P(BAAB) = b.a.a.b = a^2b^2 = $(1/N)^2$ $(N-1/N)^2$

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Probability that two out of four data hash to the same address?

Find all the possible events:
BBAA,   BAAB, BABA, AABB, ABBA, ABAB

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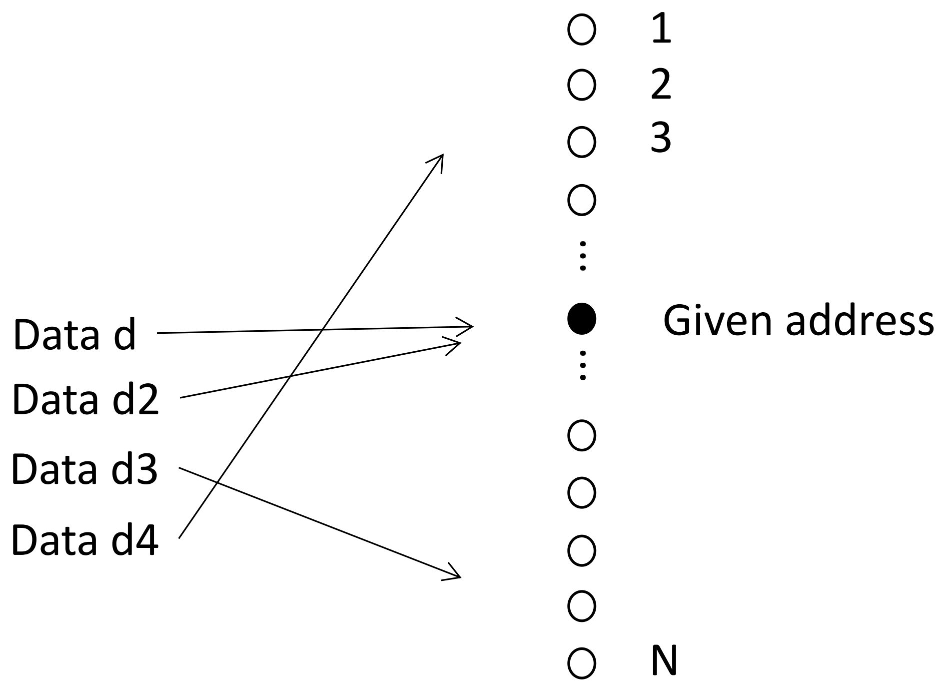

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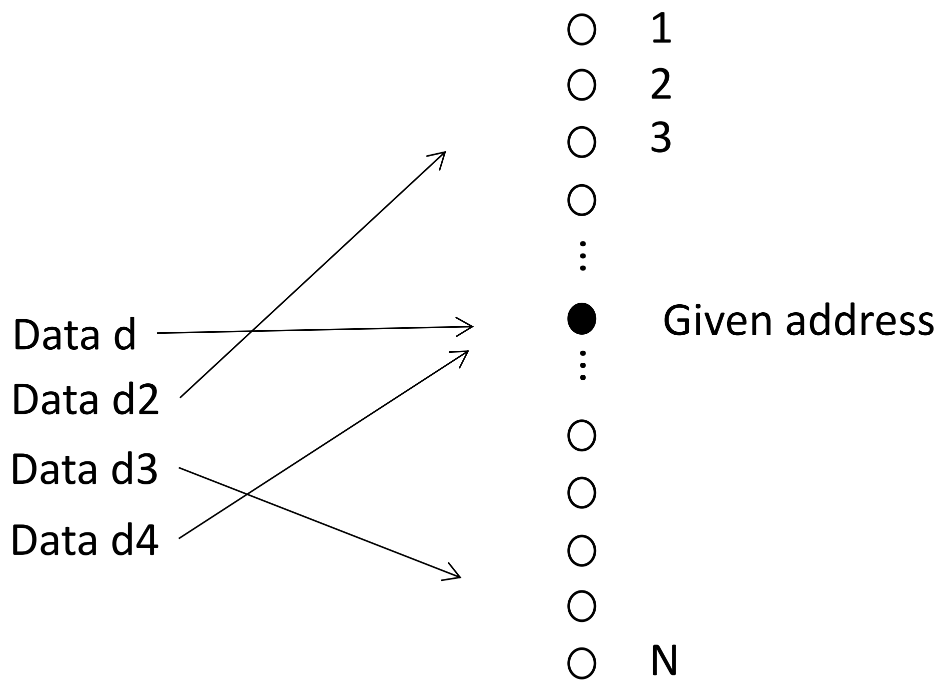

Probability that two out of four data hash to the same address?

Find all the possible events:
BBAA, BAAB, BABA, AABB, ABBA, ABAB

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Probability that two out of four data hash to the same address?

Find all the possible events:
AABB, BAAB, BABA, BBAA, ABBA, ABAB

P= P(AABB) + P(BAAB) + ... = 6 a2b2
P= C_4^2 a^2 b^2.

C_4^2 represents the number of ways two A's (and two B's) can be placed in four slots.

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Generalisation
Probability that x out of r data hash to the same address?

x times the event B and (r-x) times the event A

All the possible events : C_r^x

$$P(x) = C_r^x \cdot a^{r-x} \quad b^x$$

with $C_r^x = r! / (x! (r-x)!)$

This also means: Probability that a given address is chosen x times and not chosen r-x times.

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Calculation:
If N possible addresses
$P(x) = C_r^x \; a^{r-x} \; b^{x} = P(x) = C_r^x \; (1-1/N)^{r-x} \; (1/N)^{x}$

P(x=0) means the probability that a given address is never chosen.
$P(0) = C_r^0 \, (1-1/N)^r \, (1/N)^0$

P(x=1) means the probability that a given address is chosen only once.$P(1) = C_r^1 \, (1-1/N)^{r-1} \, (1/N)^1$

Drawback of the formula: difficult to calculate for large N and r.
The POISSON function is a good approximation.

$P(x) \; =/ \; f(x) = (\; (r/N)^{x} \, . \; e^{-(r/N)} \;) \; / \; x!$

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Synthesis

If N is the number of possible addresses, r is the number of inserted data, and x is the number of data with the same address (x times event B and r-x times event A in C_r^x possible ways),

P(x) gives the probability that x data out of r inserted ones hash to the same address.

P(x) Probability that a given address is chosen x times and not chosen r-x times.

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Synthesis

If N is the number of possible addresses, r is the number of inserted data, and x is the number of data with the same address (x times event B and r-x times event A in C_r^x possible ways),

P(x) is also the proportion of addresses with x data assigned to them by hashing.

N.P(x) is the number of addresses that have x data assigned.

This allows us to predict the number of collisions (overflow data).

Formula : N (P(2) + 2P(3) + ...iP(i+1) + ...)

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Collision prevention

Let's consider N=10,000 possible addresses and r=10,000 inserted data.

What is the number of addresses that have no data assigned?
10,000 P(0) = 3,679

What is the number of addresses that have only one data assigned?
10,000 P(1) = 3,679

What is the number of addresses that have only two data assigned?
10,000 P(2) = 1,839
→ 1,838 will be in overflow.

What is the number of addresses that have only three data assigned?
10,000 P(3) = 613
→ 613 * 2 will be in overflow.

Not ideal distribution : we have thousands of addresses (3,679) with no data assigned.

More than 1839 + 1226 (= 2 * 613) data will be in overflow.

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Collision reduction

Let's demonstrate, using examples, how,

on the one hand,
increasing the number of possible addresses

and, on the other hand,
using boxes,

can reduce collisions.

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Increase of the address space

We define the density d as follows: d = r / N (r: number of stored data; N: number of possible addresses).

Let's examine the behavior of hash functions (collisions) for different values of d.

$$P(x) = \left((r/N)^{x} \; e^{-(r/N)} \right) / x! = \left(d^{x} \,.\, e^{-d} \right) / x!$$

P(x) depends on the ratio r/N, that is, on d.

Also, we observe the same behavior for 500 data distributed among 1000 addresses as for 500,000 data distributed among 1 million addresses (d = 50% for both cases).

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Increase of the address space

Let's take d = 0.5 (N = 1000 and r = 500 data) with P(0) = 0.607; P(1) = 0.303; P(2) = 0.0758; P(3) = 0.0126; P(4) = 0.0016; etc.

How many addresses will have 0 data assigned?
1,000 * P(0) = 607

How many addresses will have 1 data assigned?
1,000 * P(1) = 303

How many addresses will have 2 data assigned?
1,000 * P(2) = 76

How many addresses will have at least two data assigned?
→ 1,000 * (P(2) + P(3) + P(4) + ...) = 90

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Increase of the address space

What is the number of data in overflow?
→ 1,000 * (P(2) + 2 * P(3) + 3 * P(4) + 4 * P(5) ......) = 107

What is the percentage of data in overflow?
→ 107/500 = 21.4%

Conclusion:
If the density is 50%, we can expect 78.6% of data stored in their primary address and 21.4% stored elsewhere.

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Use of the boxes (b > 1)

We accept b data per possible address.

In this case: $d = r / (b.N)$

where:
b: number of data per slot
N: number of addresses
r: number of inserted data.

	b=1	b=2
Number of data (r)	750	750
Number of addresses (N)	1000	500
Density (d)	0.75	0.75
Ratio (r/N)	0.75	1.5

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Use of the boxes (b > 1)

P(x)	b=1 (r/N = 0.75)	b=2 (r/N = 1.5)
P(0)	0.423	0.223
P(1)	0.354	0.335
P(2)	0.113 (Collisions)	0.251
P(3)	0.033 (Collisions)	0.126 (Collisions)
P(4)	0.006 (Collisions)	0.047 (Collisions)

Number of data in overflow in each case?

b=1:
1 000.[P(2) + 2.P(3) + 3.P(4) + ...]> 197

b=2
5 00.[P(3) + 2.P(4) + 3 P(5) + ...] > 110

The larger the box, the better the performance.